# Physics with Maple V: Collisions, Differentiation, Integration



## I. Objective

To learn the basic symbolic algebra and calculus operations of **Maple**.

## II. Introduction

For the past two weeks we have been solving problems numerically. However, much of what we do in physics involves symbolic manipulation of formulas. For this reason, programs called **symbolic solvers** are useful. These programs are capable of performing symbolic mathematical operations. We will use a symbolic solver on Project Vincent called **Maple V**. To begin a Maple session type **add maple** at the Vincent prompt, then type **xmaple** if you are in a **X-Windows** environment.

When possible, we must critically examine the results of a Maple session to ensure that it makes sense. Maple performs the mathematics so quickly that it is easy to fall into a pattern of believing every result. (You might have entered an incorrect command or Maple might interpret a command different than you do.) It is best to approach the results critically, and *assume the result is wrong until proven correct.* We will discuss methods of checking solutions in the exercises.

Because Maple is best learned by example, the exercises contain a few examples of the commands you will learn today. For more examples and information about these commands, enter the help command by typing **?** or the Maple tutorial by typing **tutorial();** at the Maple prompt.

## III. Exercises

### A. Examples: Syntax, Algebra, and Graphs

| | |
|---|---|
| Objective: | to use Maple commands to solve a physics problem. |
| Where to begin: | start in Maple V on Project Vincent. |
| What to do: | symbolically solve the mechanics problem described below. Follow the steps presented below. |
| What to turn in to your instructor: | graph described below. |
| What to put in log book: | the time you begin your work, problems, solutions, new commands, etc. |

(1) **Physics 221 Mechanics Problem:** We begin with a one dimensional collision to show Maple's ability to perform algebra. Suppose mass $m$ moving initially with speed $v_0$ collides undergoes a one dimensional collision with a mass $M$ initially at rest.


**Before Collision** **After Collision**

We assume that the collision is elastic, meaning that the total kinetic energy of the system is conserved. We also know that the total momentum is conserved, and therefore one can write:

$$mv_0 = mv + MV$$

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2$$

Because we have two equations, we may solve the above system for any two unknowns. As an example, we will solve for $v$ and $V$, and create a graph.

(2) **Basic Syntax, Line Termination, Assigning Variables:** We will now enter these equations in Maple. To make our work easier, it is convenient to assign names to the equations, for example eq1 and eq2. To assign a name to an object use the symbol **:=**. Enter the equations in Maple as shown below:

**eq1:=m*v0=m*v+M*V;**

**eq2:=1/2*m*v0^2=1/2*m*v^2+1/2*M*V^2;**

Note the following punctuation symbols and their meanings:

| Symbol | What it does |
|---|---|
| eq1:= | assigns the name **eq1** to the equation to the right. (Note that the := symbol may be used to assign names to other objects, such as expressions, variables, lists, etc.) |
| ; | terminates command lines: tells Maple to return the output |
| : | terminates command lines: tells Maple to suppress output (try it above and see) |

Note that Maple is **case sensitive**: It distinguishes between upper and lower case letters.fs

(3) **Solving Sets of Simultaneous Equations:** Now we will solve for $v$ and $V$. Enter the command below:

**sol:=solve({eq1,eq2},{v,V});**

The symbols are explained below:

| Symbol | What it does |
|---|---|
| sol:= | assigns the name **sol** to the object on the right-hand side. |
| solve({eq},{var}) | begins Maple's routine for solving the system of equations (whose names are listed in the first set of curly brackets) for the specified variables (whose names are listed in the second set of curly brackets). The solution is returned. |
| {eq1,eq2} | this is the list of equation names in our problem (note all lists in Maple are specified with curly brackets) |
| {v,V} | this is the list of variable names in our problem |

Maple will return the output below:

$$sol := \{v = v0, V = 0\}, \left\{v = \frac{(m-M)v0}{(m+M)}, V = \frac{2mv0}{(m+M)}\right\}$$

Because one equation was quadratic, there are two possible solutions, which Maple presents in two lists (curly brackets are for lists). The first solution corresponds to the time before the collision since the speeds are the same as the initial conditions, so we exclude it. The second solution is what we want.

(4) **Manipulating Solutions:** We wish to choose the second solution and to assign the solutions found to the variables $v$ and $V$. To do this, we type

**assign(sol[2]);**

Note: The object **sol** is a *list* of two equations. The object **sol[2]** is the second one of these two equations. If **eq** is an equation of the form **x=expr**, then **assign(eq)** assigns the expression **expr** to the variable **x**. Let's see how this worked: In Maple, type

**v;**

**V;**

and observe the ouput. Note that the meanings of the variables $v$ and $V$ have changed. They are no longer independent variables, but have been replaced by their values as a function of $m$, $M$, and $v_0$. The equations **eq1** and **eq2** have also changed, since the solutions $v$ and $V$ are now plugged into the equations. What happens if you type

**eq1;**

**eq2;**

(5) **Checking Solutions:** Maple provides a way of checking solutions by substituting the solutions back into the original equation. Because the solving routine and checking routine are separate programs, it is highly unlikely that bugs exist in both that would cancel each other out. Therefore, this check is a meaningful way of gaining confidence in your solution. Execute the command below and note the result:

**simplify({eq1,eq2});**

$$\left\{\tfrac{1}{2}mv0^2 = \tfrac{1}{2}mv0^2, mv0 = mv0\right\}$$

The variables were substituted back into the equations and the result is a list of two identities. Since both sides of each equation are the same, our formulas really are solutions to the equations we specified.

| Symbol | What it does |
|---|---|
| simplify( ) | simplifies algebraic expressions |

(6) **Checking Limiting Cases:** Another way of checking our results is to check the formulas in the limits of its variables and compare the result with what you expect from physical intuition. For example, what do you expect the speed of mass $m$ to become in the limit $M \to \infty$? To take this limit type:

**limit(v,M=infinity);**

What output does Maple give in the case $m=M$? the case $m \to \infty$? Are these results consistent with what you expect?

(7) **Making a 2D Plot:** Suppose now we wish to graph the behavior of $v$ as a function of the mass ratio *M/m.* We first have to write the formula

$$v = \frac{m - M}{m + M} v_0$$

in a form that has no units. One way is to plot the ratio $y=v/v_0$ on the vertical axis and the ratio $x=M/m$ on the horizontal axis. Using Maple, we can show that

$$y = \frac{1 - x}{1 + x}$$

To define $y$ and then write $M$ in terms of $x$ in Maple type:

**y:=v/v0:**
**M:=m*x:**
**y;**

$$\frac{m - xm}{m + xm}$$

Now simplify the last expression.
**simplify(“);**

$$-\frac{-1 + x}{1 + x}$$

| Symbol | What it does |
|---|---|
| “ | last expression |
| simplify(“) | simplifies the previous output. We could also use **simplify(y)** to get what we want. |

To make the plot, type:

**plot(y,x=0..10);**

Save your graph by going to the **Print** submenu of the **File** menu on the graph window. Select the file type **Postscript** and give your file a name. To print out the graph type the following command at the vincent prompt: **lpr** ***filename***. Your graph will be sent to the laser printer in room 139 Durham, and will be put in the BIN defined in your login sequence. Type **env | more** to find your bin number.

(8) **Save your work:** Use the **File** pull down menus to **Save** your **sess1.ms**. Once your session is saved, you may open it later using the **Open** command in the **File** menu. You may also **Print** it (from the **File** menu) to a postscript file, and then send the file to the printer using the **lpr** command at the Vincent prompt.

## B. Application to Ballistic Pendulum

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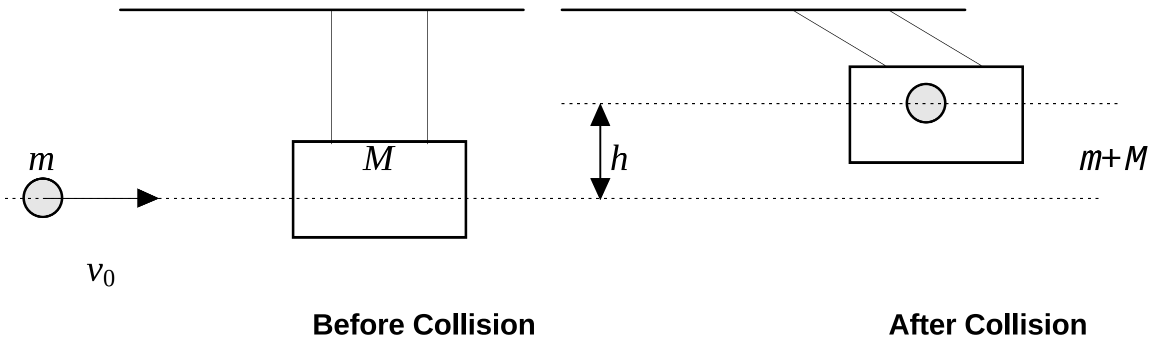
|---|---|
| Objective: | use Maple to solve a ballistic pendulum problem |
| Where to begin: | in Maple V (choose the command **New** under the **File** menu to begin a new session) |
| What to do: | obtain and check your solution using methods presented in the previous example |
| What to turn in to your instructor: | (1) a copy of your Maple session; (2) graph described below |
| What to put in log book: | the time you begin your work, problems, solutions, new commands, etc. |

(1) **Ballistic Pendulum:** You have all worked through this problem before, probably in the PHY 221 lab. A ballistic pendulum is a large block of wood that is free to swing. A bullet is fired into the block, and by using the conservation of energy and linear momentum one may determine the initial speed of the bullet $v_0$ by measuring the maximum height $h$ to which the block rises (see the figure below).




The conservation laws yield the equations

$$mv_0 = (m+M)V$$

$$\tfrac{1}{2}(m+M)V^2 = (m+M)gh$$

where $V$ is the speed of the block immediately after the bullet is imbedded.

(2) **Solve the Equations:** Following the methods of the previous example, solve the system for $V$ and $h$. Be sure check the result by substituting the solution back into the equations.

(3) **Check Limiting Cases:** Verify that the expressions you found for $h$ and $V$ are correct in the three limiting cases $m \to 0$, $M \to \infty$ , $v_0 \to 0$.

(4) **Graph Results:** Generate a plot of your result for $h$ (you will have to rescale it to make it unitless) versus the variable $M/m$ over the range from 0 to 10.

(5) **Saving and Printing:** Save your session as **sess2.ms** and print out both the text of the session and the graph to submit to your instructor.

## C. Derivatives

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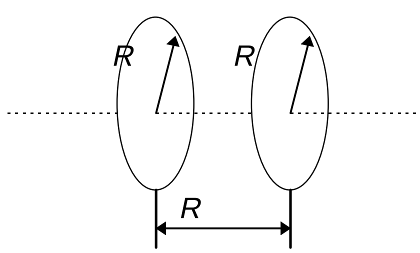
|---|---|
| Objective: | use Maple to solve a problem involving derivatives. |
| Where to begin: | in Maple V (choose the command **New** under the **File** menu to begin a new session). |
| What to do: | use the differentiation command **diff** to solve the problem below. |
| What to turn in to your instructor: | a copy of your Maple session. |
| What to put in log book: | the time you begin your work, problems, solutions, new commands, etc. |

(1) **Examples:** To differentiate an expression use the **diff** command. First assign the expression to a variable (see Exercise A), for example the variable **func**. Differentiation with respect to a variable is then done as shown below:

| Maple Command | Mathematical Operation |
|---|---|
| **diff(func,x);** | $\frac{\partial}{\partial x} func$ |
| **diff(func,x$2);** | $\frac{\partial^2}{\partial x^2} func$ |
| **diff(func,x$2,y);** | $\frac{\partial}{\partial y}\frac{\partial^2}{\partial x^2} func$ |

(2) **Helmholtz Coils:** Helmholtz coils are a set of two circular coils of radius $R$ whose centers are a distance $R$ apart. A steady current $I$ flows in the same direction around each coil as shown in the figure below. The magnetic field *on the axis* at a distance $x$ from the center of one coil is

$$B = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(2R^2+x^2-2Rx)^{3/2}}\right]$$




(3) **Determine Derivatives:** Find the value of B, $\frac{dB}{dx}$, and $\frac{d^2B}{dx^2}$ at the midpoint between the two coils x=*R/2* (note that the command **x:='x'** will clear the variable x of its previous values).

**Hints:**

Start by typing **B(x):=....;** to enter the define the function $B(x)$.

To evaluate this function or the derivatives at a particular value of $x$ (say $x=R/2$), use the **subs()** function: **subs(x=R/2,B(x));**

## D. Integrals

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|---|---|
| Objective: | use Maple to solve a problem involving integrals |
| Where to begin: | in Maple V (choose the command **New** under the **File** menu to begin a new session) |
| What to do: | use the integration command **int** to solve the problem below |
| What to turn in to your instructor: | a copy of your Maple session |
| What to put in log book: | the time you begin your work, problems, solutions, new commands, etc. |

(1) **Examples:** To integrate an expression use the **int** command.

| Maple Command | Mathematical Operation |
|---|---|
| **int(func,x);** | $\int dx func$ |
| **int(int(func,x),y);** | $\int dy \int dx func$ |
| **int(func,x=a..b);** | $\int_a^b dx func$ |
| **int(int(func,x=a..b),y=c..d);** | $\int_c^d dy \int_a^b dx func$ |

(2) **Problem:** The electric field due to a thin, uniformly charged, circular disk of charge $Q$ and radius $R$ at a point on the symmetry axis a distance $z$ from the disk center can be written as

$$\vec{\mathbf{E}} = \hat{\mathbf{z}} \frac{z\sigma}{4\pi\varepsilon_0} \int_0^{2\pi} d\phi \int_0^R \frac{rdr}{(z^2 + r^2)^{3/2}}$$

where $\sigma = Q/(\pi R^2)$ is the surface charge density. Evaluate the expression and obtain a formula for the magnitude of the electric field. Note that the symbol **E** is a forbidden variable name because Maple assigns it a special value. Also, the constant $\pi$ =3.14... is specified in Maple by the variable **Pi** (first letter capitalized).-

(3) **Check a Limiting Case:** In the case that the disk becomes infinitely large, one expects that the electric field should approach that of a uniformly charged, infinite plane which is given by $E_{plane} = \sigma / 2\varepsilon_0$. Verify this.

## IV. Appendix 1: Basic Maple Commands[1] (text version)

| Category | Command | What it does |
|---|---|---|
| **Help** | ?*topic* | gives help on a *topic* |
| **Tutorial** | tutorial( ); | begins the Maple V tutorial session |
| **Session Commands** (without mouse) | restart; | clears all variables and begins a new session; |
| | save `*sess.**m***` | saves a Maple session to file *sess.**m*** (note that the ***.m*** extension is necessary) |
| | read `*sess.**m***` | reads a previously saved Maple session |
| **Algebra Commands** | simplify( ); | simplifies algebraic expressions |
| | factor( ); | factors algebraic expressions |
| | expand( ); | expands algebraic expressions |
| | solve( ); | solves specified system of equations for specified unknowns |
| | subs( ); | substitutes specified variables into specified equations |
| | assign( ); | assigns solutions to variable names |
| | evalf( ); | numerically evaluates a function |
| **Calculus Commands** | diff( ); | differentiates function |
| | int( ); | integrates function |
| | limit( ); | takes the limit of a function |
| | series( ); | performs a series expansion of the function |
| **Graphics Commands** | plot( ); | performs a 2D plot |
| | plot3d( ); | performs a 3D plot |

## V. Appendix 2: Saving and Printing in the X-Maple Environment

The table below gives instructions on how to perform simple operations when using **X-Maple**, the X-Windows version of Maple V.

| | |
|---|---|
| **How to Save Maple Session** | pull down the **File** menu and select **Save** |
| **How to Open Maple Session** | pull down the **File** menu and select **Open** |
| **How to Print Out a Maple Session** | (1) pull down the **File** menu and select the **Print** option<br>(2) print the current session to a **Postscript** file<br>(2) at the Vincent prompt, use<br>**lpr -P**(printer name) **-Dpostscript *filename***<br>command to print out file (or simply **lpr filename**). |
| **How to Print Out a Graph** | (1) in the graph window, pull down the **File** menu;<br>(2) choose the option **Print** and pick the format **Postscript**;<br>(3) at the Vincent prompt, use<br>**lpr -P**(printer name) **-Dpostscript *filename***<br>to print the file (or simply **lpr filename**). |

[1]Note that this table gives the command name, but not the syntax. For syntax examples see the section Exercises, or use the help command (type ?*command* at the Maple prompt).